Finding the rates inside a PEPA model with EMPEPA

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PEPA Club

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Outline

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Motivation

Phase-type distributions & EM algorithm

EMPEPA, finding the rates inside a PEPA model

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- Motivation
- Phase-type distributions & EM algorithm
- EMPEPA, finding the rates inside a PEPA model
- Conclusion

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1. To get a realistic PEPA model of a distributed system.

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2. Choosing the transition rates by using the technical leaflet of the hardware does not work well.

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Motivation

1. To get a realistic PEPA model of a distributed system.
2. Choosing the transition rates by using the technical leaflet of the hardware does not work well.
3. $\Rightarrow$ Massive hand tunning,
4. better do automatic tuning!

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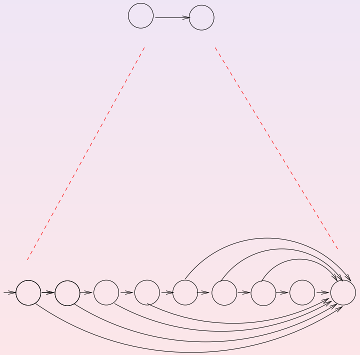
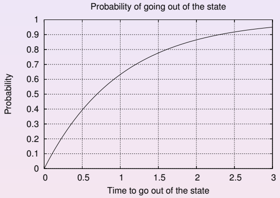
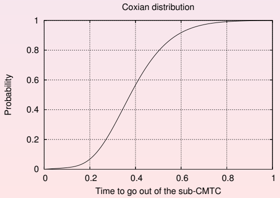
- Motivation
- Phase-type distributions & EM algorithm
- EMPEPA, finding the rates inside a PEPA model
- Conclusion

Phase-type distributions (Neuts 1981)

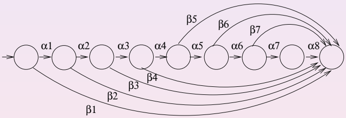


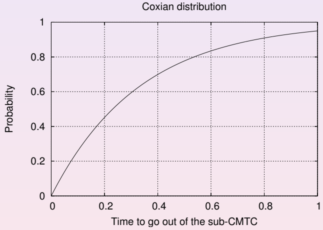




Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.





$$\alpha_1 = 0.01,\ \alpha_2 = 0.01,\ \alpha_3 = 0.1,\ \alpha_4 = 10,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 3,\ \beta_2 = 2,\ \beta_3 = 1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

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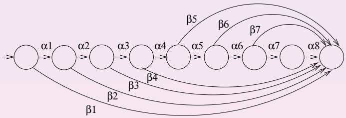
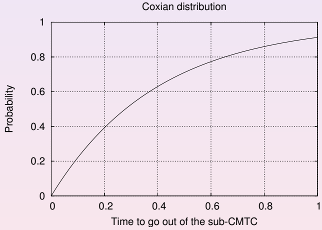


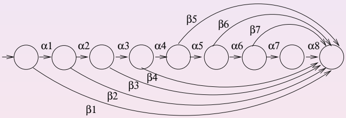


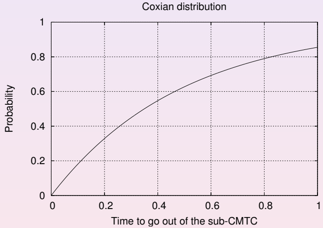
$$\alpha_1 = 0.05,\ \alpha_2 = 0.1,\ \alpha_3 = 10,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 2.5,\ \beta_2 = 1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



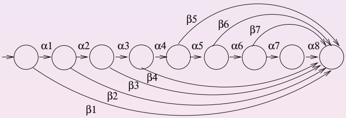


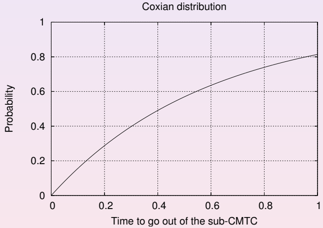
$$\alpha_1 = 0.1,\ \alpha_2 = 0.1,\ \alpha_3 = 0.1,\ \alpha_4 = 10,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 2,\ \beta_2 = 1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



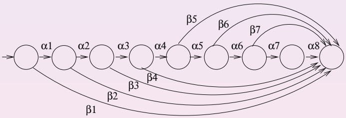


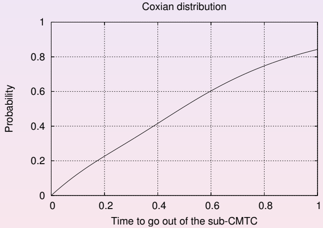
$$\alpha_1 = 1,\ \alpha_2 = 10,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1.7,\ \beta_2 = 1,\ \beta_3 = 0.1,\ \beta_4 = 0.01,\ \beta_5 = 0.01,\ \beta_6 = 0.01,\ \beta_7 = 0.01$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



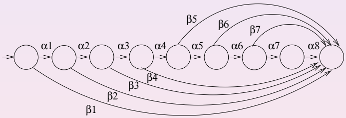


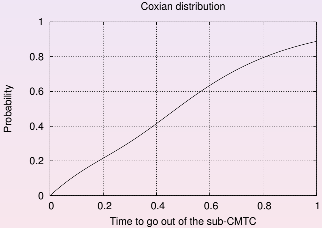
$$\alpha_1 = 1,\ \alpha_2 = 10,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1.4,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.01,\ \beta_5 = 0.01,\ \beta_6 = 0.01,\ \beta_7 = 0.01$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



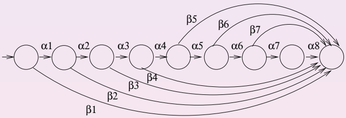


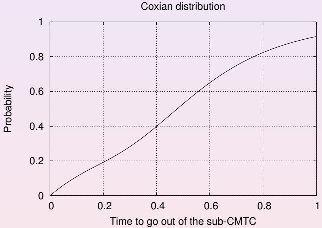
$$\alpha_1 = 1.7,\ \alpha_2 = 11,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1.4,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.





$$\alpha_1 = 2.5,\ \alpha_2 = 11,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1.3,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

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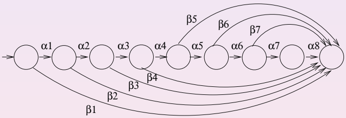
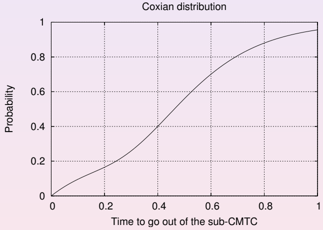


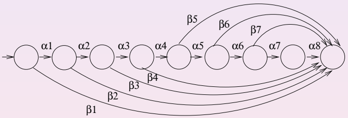


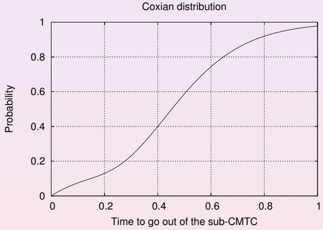
$$\alpha_1 = 4,\ \alpha_2 = 12,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1.2,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



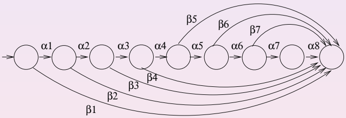


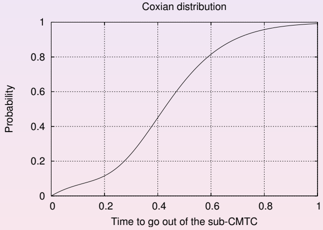
$$\alpha_1 = 6,\ \alpha_2 = 12,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 1,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



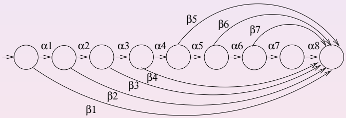


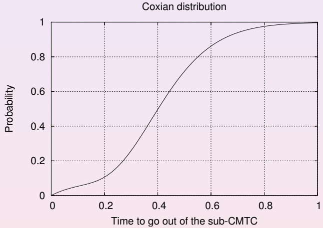
$$\alpha_1 = 8,\ \alpha_2 = 16,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 0.9,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.



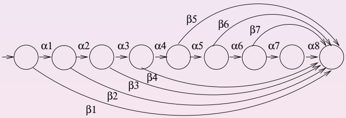


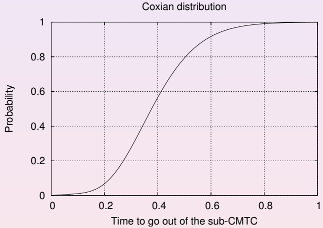
$$\alpha_1 = 10,\ \alpha_2 = 20,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 0.8,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

Phase type distributions (Neuts 1981)

Transition rates $\Longrightarrow$ shape of the distribution.

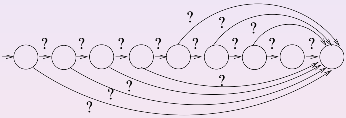


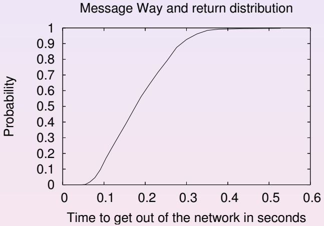


$$\alpha_1 = 20,\ \alpha_2 = 20,\ \alpha_3 = 20,\ \alpha_4 = 20,\ \alpha_5 = 20,\ \alpha_6 = 20,\ \alpha_7 = 20,\ \alpha_8 = 20$$

$$\beta_1 = 0.1,\ \beta_2 = 0.1,\ \beta_3 = 0.1,\ \beta_4 = 0.1,\ \beta_5 = 0.1,\ \beta_6 = 0.1,\ \beta_7 = 0.1$$

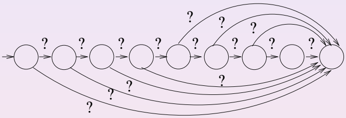
In reality non-exponential distributions

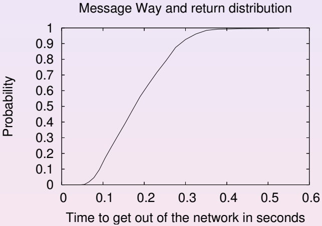




- How to find the rates such that it describes a given distribution?

In reality non-exponential distributions





- How to find the rates such that it describes a given distribution?
- Using the EM algorithm (Dempster 1977) for phase-type distributions (Asmussen et al. 1996)

EM algorithm

- Partially observable system (hidden state variables).

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- Iterative method :
  1. E step for *Expectation*
  2. M step for *Maximization*.

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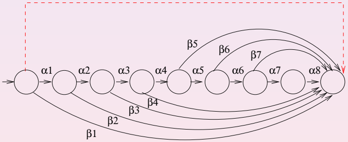
EM algorithm

- Partially observable system (hidden state variables).
- Iterative method :
  1. E step for *Expectation*
  2. M step for *Maximization*.
- Initialization of the algorithme : the user provide an initial model supposed to explain the observations.
- At each iteration the algorithm refines the model until convergence (possibly local).

EM for fitting phase-type distributions (Asmussen et al. 1996)

- The observable part : response time of the systme,

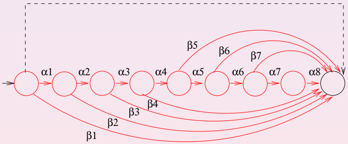


$t = 32ms$
$t = 45ms$
$t = 12ms$
$t = 19ms$
$t = 7ms$
.
.
.

Figure: *Model (left), partial observations (right)*

EM for fitting phase-type distributions (Asmussen et al. 1996)

- The observable part : response time of the systme,
- The hidden part : time spent in each state and number of times each transition is taken.

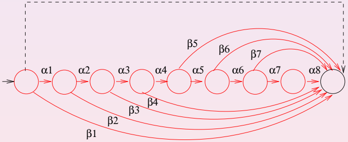


$t_{s_1} = 2ms$	$t_{s_2} = 1ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 4ms$	$t_{s_2} = 4ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 12ms$	$t_{s_2} = 0ms$	$\dots$	$n_{\alpha_1} = 0$	$\dots$
$t_{s_1} = 8ms$	$t_{s_2} = 6ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 7ms$	$t_{s_2} = 0ms$		$n_{\alpha_1} = 0$	
$\vdots$	$\vdots$		$\vdots$	

Figure: *Model (left), complete observations (right)*

EM for fitting phase-type distributions (Asmussen et al. 1996)

- The observable part : response time of the systme,
- The hidden part : time spent in each state and number of times each transition is taken.



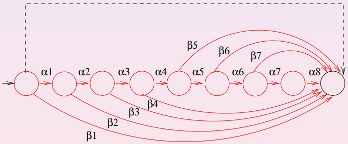
$t_{s_1} = 2ms$	$t_{s_2} = 1ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 4ms$	$t_{s_2} = 4ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 12ms$	$t_{s_2} = 0ms$	$\dots$	$n_{\alpha_1} = 0$	$\dots$
$t_{s_1} = 8ms$	$t_{s_2} = 6ms$		$n_{\alpha_1} = 1$	
$t_{s_1} = 7ms$	$t_{s_2} = 0ms$		$n_{\alpha_1} = 0$	
$\vdots$	$\vdots$		$\vdots$	

Figure: *Model (left), complete observations (right)*

E step : estimation of the hidden variables $\Longrightarrow$ ODEs

EM for fitting phase-type distributions (Asmussen et al. 1996)

- The observable part : response time of the systme,
- The hidden part : time spent in each state and number of times each transition is taken.



$t_{s_1} = 2ms$ $t_{s_1} = 4ms$ $t_{s_1} = 12ms$ $t_{s_1} = 8ms$ $t_{s_1} = 7ms$ $\vdots$	$t_{s_2} = 1ms$ $t_{s_2} = 4ms$ $t_{s_2} = 0ms$ $t_{s_2} = 6ms$ $t_{s_2} = 0ms$ $\vdots$	...	$n_{\alpha_1} = 1$ $n_{\alpha_1} = 1$ $n_{\alpha_1} = 0$ $n_{\alpha_1} = 1$ $n_{\alpha_1} = 0$ $\vdots$	...

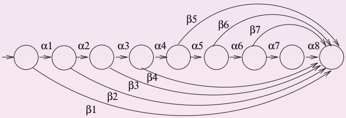
Figure: *Model (left), complete observations (right)*

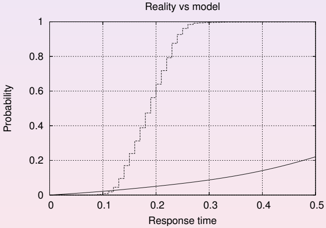
E step : estimation of the hidden variables $\Longrightarrow$ ODEs

M step : $\textit{transition rate} = \dfrac{\textit{number of times the transition is taken}}{\textit{time spent in the state preceding the transiton}}$

EM algorithm : example

Iteration 0-initialization :



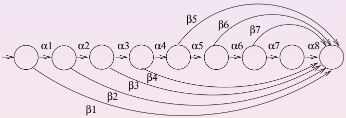


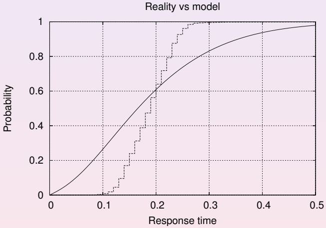
$$\alpha_1 = 10,\ \alpha_2 = 15,\ \alpha_3 = 7,\ \alpha_4 = 12,\ \alpha_5 = 11,\ \alpha_6 = 10,\ \alpha_7 = 8,\ \alpha_8 = 6$$

$$\beta_1 = 0.2,\ \beta_2 = 0.15,\ \beta_3 = 0.45,\ \beta_4 = 0.31,\ \beta_5 = 0.22,\ \beta_6 = 0.4,\ \beta_7 = 0.3$$

EM algorithm : example

Iteration 1 :



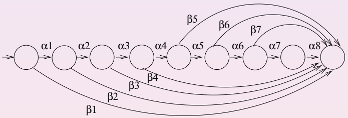


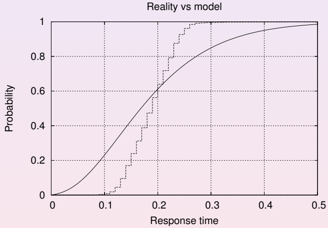
$$\alpha_1 = 12.7,\ \alpha_2 = 19.1,\ \alpha_3 = 6.38,\ \alpha_4 = 15.3,\ \alpha_5 = 21.3,\ \alpha_6 = 21.3,\ \alpha_7 = 28.6,\ \alpha_8 = 32.3$$

$$\beta_1 = 1.5,\ \beta_2 = 2.1,\ \beta_3 = 9.8,\ \beta_4 = 1.12,\ \beta_5 = 9.12,\ \beta_6 = 11.4,\ \beta_7 = 5.1$$

EM algorithm : example

Iteration 2 :



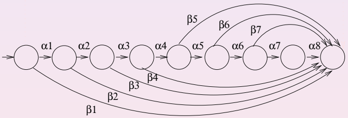


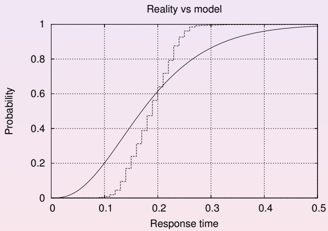
$$\alpha_1 = 13.7,\ \alpha_2 = 19.4,\ \alpha_3 = 6.67,\ \alpha_4 = 15.7,\ \alpha_5 = 21.3,\ \alpha_6 = 21.3,\ \alpha_7 = 36,\ \alpha_8 = 44.7$$

$$\beta_1 = 0.54,\ \beta_2 = 1.4,\ \beta_3 = 10.8,\ \beta_4 = 14.8,\ \beta_5 = 14,\ \beta_6 = 18.6,\ \beta_7 = 8.52$$

EM algorithm : example

Iteration 4 :



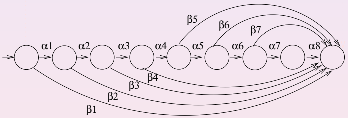


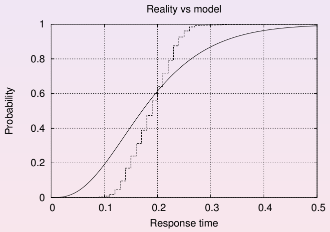
$$\alpha_1 = 15.7,\ \alpha_2 = 19.3,\ \alpha_3 = 8.34,\ \alpha_4 = 17.4,\ \alpha_5 = 26.9,\ \alpha_6 = 26.1,\ \alpha_7 = 50.4,\ \alpha_8 = 66.7$$

$$\beta_1 = 0.015,\ \beta_2 = 0.36,\ \beta_3 = 10,\ \beta_4 = 17.8,\ \beta_5 = 21,\ \beta_6 = 30.8,\ \beta_7 = 15.4$$

EM algorithm : example

Iteration 8 :



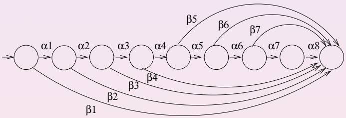


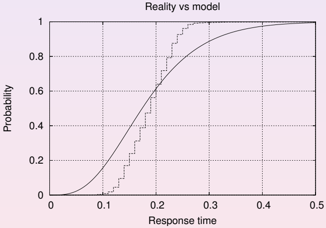
$\alpha_1 = 17.1,\ \alpha_2 = 19.1,\ \alpha_3 = 10.4,\ \alpha_4 = 19.1,\ \alpha_5 = 29,\ \alpha_6 = 27.6,\ \alpha_7 = 57.9,\ \alpha_8 = 79.8$

$\beta_1 = 0.0002,\ \beta_2 = 0.07,\ \beta_3 = 8.2,\ \beta_4 = 17.3,\ \beta_5 = 23.5,\ \beta_6 = 37.1,\ \beta_7 = 19.9$

EM algorithm : example

Iteration 16 :



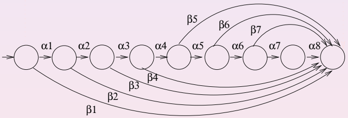


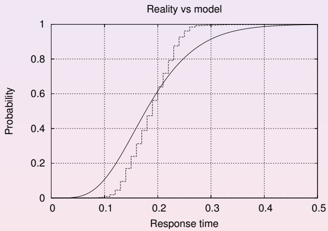
$\alpha_1 = 20.8,\ \alpha_2 = 21.1,\ \alpha_3 = 18.7,\ \alpha_4 = 23.3,\ \alpha_5 = 32.6,\ \alpha_6 = 30.4,\ \alpha_7 = 68.8,\ \alpha_8 = 99.4$

$\beta_1 = 0.0,\ \beta_2 = 0.0002,\ \beta_3 = 2.37,\ \beta_4 = 10.5,\ \beta_5 = 21,\ \beta_6 = 41.5,\ \beta_7 = 25.7$

EM algorithm : example

Iteration 32 :



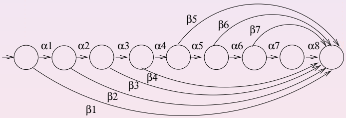


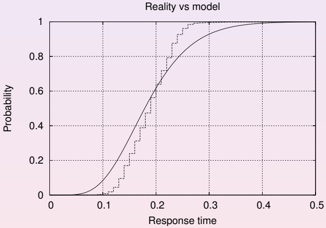
$$\alpha_1 = 27.7,\ \alpha_2 = 27.7,\ \alpha_3 = 27.7,\ \alpha_4 = 29.1,\ \alpha_5 = 33.7,\ \alpha_6 = 33.4,\ \alpha_7 = 70.1,\ \alpha_8 = 100$$

$$\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_3 = 0.002,\ \beta_4 = 0.7,\ \beta_5 = 6,\ \beta_6 = 25.9,\ \beta_7 = 21.6$$

EM algorithm : example

Iteration 48 :



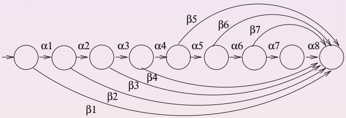


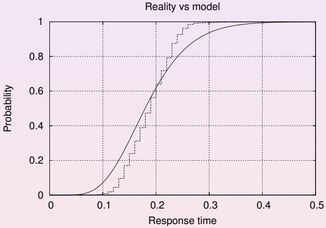
$$\alpha_1 = 33,\ \alpha_2 = 33,\ \alpha_3 = 33,\ \alpha_4 = 33.4,\ \alpha_5 = 35.2,\ \alpha_6 = 35.6,\ \alpha_7 = 62.8,\ \alpha_8 = 86.4$$

$$\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_3 = 0.0,\ \beta_4 = 0.001,\ \beta_5 = 0.52,\ \beta_6 = 9.36,\ \beta_7 = 12.8$$

EM algorithm : example

Iteration 64 :



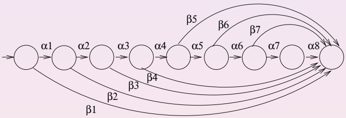


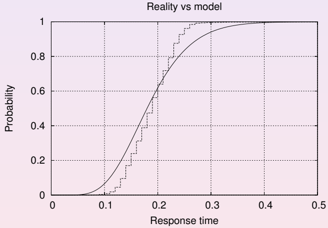
$$\alpha_1 = 37,\ \alpha_2 = 37,\ \alpha_3 = 37,\ \alpha_4 = 37.1,\ \alpha_5 = 37.6,\ \alpha_6 = 38.5,\ \alpha_7 = 52.9,\ \alpha_8 = 68.7$$

$$\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_3 = 0.0,\ \beta_4 = 0.0,\ \beta_5 = 0.005,\ \beta_6 = 1.7,\ \beta_7 = 5.7$$

EM algorithm : example

Iteration 80 :



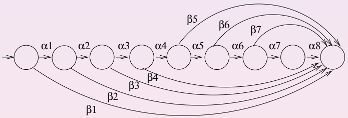


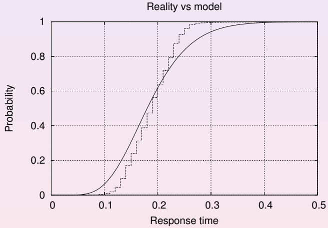
$$\alpha_1 = 39.6,\ \alpha_2 = 39.6,\ \alpha_3 = 39.6,\ \alpha_4 = 39.6,\ \alpha_5 = 39.8,\ \alpha_6 = 40.4,\ \alpha_7 = 46.3,\ \alpha_8 = 55.6$$

$$\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_3 = 0.0,\ \beta_4 = 0.0,\ \beta_5 = 0.0,\ \beta_6 = 0.15,\ \beta_7 = 2.22$$

EM algorithm : example

Iteration 100 :





$\alpha_1 = 41.5,\ \alpha_2 = 41.5,\ \alpha_3 = 41.5,\ \alpha_4 = 41.5,\ \alpha_5 = 41.5,\ \alpha_6 = 41.7,\ \alpha_7 = 43.2,\ \alpha_8 = 46.7$

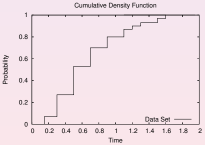
$\beta_1 = 0.0,\ \beta_2 = 0.0,\ \beta_3 = 0.0,\ \beta_4 = 0.0,\ \beta_5 = 0.0,\ \beta_6 = 0.001,\ \beta_7 = 5.22$

Outline

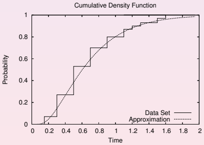
Fitting phase-type distribution with EMpht (Olsson 98)

$$
\begin{array}{l} \ldots \\ 0.499387 \\ 1.226753 \\ 0.635516 \\ 0.478954 \\ 0.403290 \\ 0.277662 \\ 0.377174 \\ \ldots \end{array}
\quad + \quad
\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix}
\xrightarrow{EMpht}
\begin{pmatrix} -4.6 & 1.3 & 2.4 & 0.3 \\ 6.2 & -8.8 & 0.2 & 3.7 \\ 3.1 & 0.4 & -5.6 & 1.2 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

$\Updownarrow$



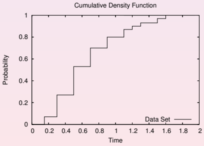
$\Updownarrow$



Fitting phase-type disitribution with EMPEPA

. . .
0.499387
1.226753
0.635516
0.478954
0.403290
0.277662
0.292342
0.377174
. . .

$\Updownarrow$



$+$

$V = \{v_1, v_2, v_3\}$

$A \stackrel{def}{=} (\alpha, v_1).(\beta, v_2).Stop$

$B \stackrel{def}{=} (\beta, \top).(\gamma, v_3).Stop$

$C \stackrel{def}{=} (\tau, v_3).(A \underset{\{\alpha,\beta\}}{\bowtie} B)$

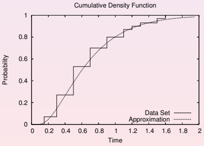
$D \stackrel{def}{=} (\tau, 4).Stop + C$

$RootTerm = D$

$\xrightarrow{EMPEPA}$

$v_1 = 3.4367$
$v_2 = 0.2647$
$v_3 = 9.6573$

$\Updownarrow$



How to do that?

1. PEPA model translated into its CTMC :

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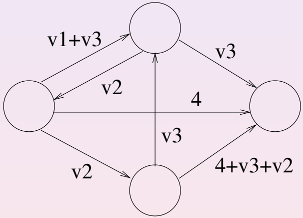
$$RootTerm = D$$

$$\Longrightarrow$$



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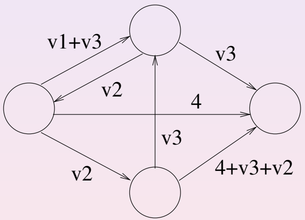

2. The M step is modified to take into account the multiple localisations of the same variable in the CTMC.

Maximizing the likelihood of a set of partially observable executions

- Generalization of the algorithm, works on PEPA models that do not describe distribution.

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  - *stop* : action of the user that stops the observation.

Using EMPEPA with partially observable executions

$$\begin{array}{l} \dots \\ init \xrightarrow{1.22} \alpha \xrightarrow{0.63} \beta \dots \xrightarrow{0.47} stop \\ init \xrightarrow{0.40} \beta \xrightarrow{0.27} \gamma \xrightarrow{0.29} stop \\ init \xrightarrow{0.37} \alpha \xrightarrow{2.31} \beta \dots \xrightarrow{0.23} stop \\ \dots \end{array} \quad + \quad \boxed{\begin{array}{l} V = \{v_1, v_2, v_3\} \\ \\ A \stackrel{def}{=} (\alpha, v_1).(\beta, v_2).B \\ B \stackrel{def}{=} (\beta, \top).(\gamma, v_3).C \\ C \stackrel{def}{=} (\tau, v_3).(A \underset{\{\alpha,\beta\}}{\bowtie} B) \\ D \stackrel{def}{=} (\tau, 4).A + C \\ \\ RootTerm = (\tau.v_2).D \end{array}} \quad \xrightarrow{EMPEPA} \quad \begin{array}{l} v_1 = 3.4367 \\ v_2 = 0.2647 \\ v_3 = 9.6573 \end{array}$$

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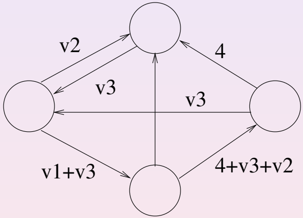
$$RootTerm = D$$

$\Longrightarrow$



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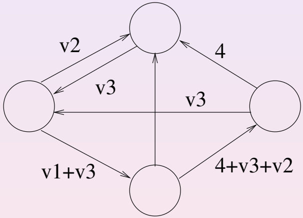

2. The *E* step has to be modified, every distribution between each couple of events has to be considered.

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\\
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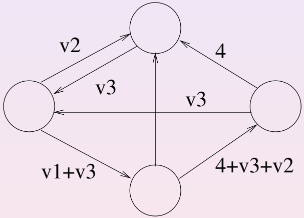
\\
RootTerm = D
\end{array}
$$

$\Longrightarrow$



2. The *E* step has to be modified, every distribution between each couple of events has to be considered.
3. The *M* step has to be modified in the same way than before.

Time for a demo!

Outline

- Motivation
- Phase-type distributions & EM algorithm
- EMPEPA, finding the rates inside a PEPA model
- Conclusion

Conclusions

- Limits :

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  - extending the algorithm for extentions of PEPA (that deals with massive parallel processes)

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  - slow...(models of a couple of hundred states)
- Possible improvements :
  - extending the algorithm for any PEPA model (any kind of transitions)
  - extending the algorithm for extentions of PEPA (that deals with massive parallel processes)
  - optimizing the algorithm : maybe by solving the fitting in the moment space (a PEPA model needs only a bounded numbers of moments to describe the distribution between two events).